Error Analysis in Mathematics

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Error analysis is a method commonly used to identify the cause of student errors when they make consistent mistakes. It is a process of reviewing a student's work and then looking for patterns of misunderstanding. Errors in mathematics can be factual, procedural, or conceptual, and may occur for a number of reasons.

Why is error analysis important?

Identification of students' specific errors is especially important for students with learning disabilities and low performing students (Fuchs, Fuchs, & Hamlett, 1994; Salvia & Ysseldyke, 2004). By pinpointing student errors, the teacher can provide instruction targeted to the student's area of need. In general, students who have difficulty learning math typically lack important conceptual knowledge for several reasons, including an inability to process information at the rate of the instructional pace, a lack of adequate opportunities to respond (i.e., practice), a lack of specific feedback from teachers regarding misunderstanding or non-understanding, anxiety about mathematics, and difficulties in visual and/or auditory processing (University of Kansas, n.d.).

Common Student Challenges

The first step of error analysis is to correctly identify the specific errors displayed in students work. First, let's look at a few reason why students may make errors.

Lack of knowledge. Students' lack of knowledge could be a major reason why they cannot solve certain problems consistently (Hudson & Miller, 2006). As noted above, there are three types of errors: procedural, factual, and conceptual (see Table 1 for specific examples). When a student has not followed the correct steps (or procedures) to solve a problem, this is a procedural error. Factual errors are mistakes that students make when they cannot recall a fact required to solve a problem or if they have not mastered basic facts (Ginsburg, 1987, as cited in University of Kansas, n.d.). Procedural and factual errors (also known as ‘slips’) are generally not due to inherent misunderstandings; slips may be due to memory deficits, impulsivity, or visual-motor integration problems and are easier to identify than conceptual errors. Conceptual errors (or ‘bugs’) may look like procedural errors, but they occur because the student does not fully understand a specific math concept, such as place value (Ginsburg, 1987, as cited in University of Kansas, n.d.). As such, bugs are more serious errors. To determine if an error is conceptual, teachers should check by asking the student to represent the problem with concrete objects or show and explain the steps used to solve the problem (Hudson & Miller, 2006).

Poor attention and carelessness. Other possible causes of student error are poor attention and carelessness (Stein, Silbert, & Carnine, 1997). To address this issue, teachers should first consider the alignment between the instruction, student ability, and the task (Hudson & Miller, 2006). For example, lack of attention is more likely to occur during a long-division lesson when the students have not learned division or mastered necessary pre-requisite skills (i.e., there is a mismatch between the instruction and student ability). Inattention could occur during parts of lesson where students are required to listen for an extended period of time. In such cases, teachers should consider delivering the materials in a brisk, enthusiastic manner, making sure that students are given ample opportunities to engage and respond to questions. When lack of attention occurs during independent work, teachers should provide clear expectations for completing tasks, monitor student work, and provide corrective feedback.

How to Conduct Error Analysis

The following steps describe the error analysis process, applied to mathematics (Howell, Fox, & Morehead, 1993):

1. Collect a sample of student work for each type of problem (e.g., single-digit addition; two-digit multiplication with regrouping), with at least three to five items for each problem type.
2. Have the student verbalize or think aloud as s/he solves the problems without providing any type of cues or prompting.
3. Record all student responses in written and verbal format.
4. Analyze the responses and look for patterns among common problem types.
5. Look for examples of "exceptions" to an apparent pattern (accurate "exceptions" could signal that the student does not fully understand the procedure or concept).
6. Describe the patterns observed in simple language and the possible reasons for the student’s problems (e.g., if a student did not regroup double-digit addition problems, it could be a sign that the student does not understand the concept of place value).
7. Interview the student by asking him/her to explain how s/he solved the problem to confirm suspected error patterns.

Table 1 provides examples with a description of the error pattern and possible causes for each error.

Table 1. Examples of Common Errors

Example	Error Pattern Description	Possible cause
493 + 28 4111	Added the ones column correctly but did not carry over the one ten to the tens column	Used inaccurate procedure (i.e procedural error)[1]
36 + 86 1112	Recorded each of the sums of the ones and tens without regrouping	A lack of regard for or non-understanding of place value[2]
346 + 39 386	Followed the correct procedures but added the ones column incorrectly	Memory deficit; have not mastered the basic facts (i.e. factual error)[1]
719 - 262 557	Subtracted the smaller number from the larger number without paying attention to the placement of the number (regardless if it is the upper number, the minuend, the lower number, or the subtrahend)	Inaccurate procedure; avoiding regrouping; misunderstanding of place value; finds regrouping difficult because of a visual/motor deficit[2]
34 + 56 18	Added all digits together	Incorrect procedure; lack of regard for place value[2]
2 1 534 + 783 1117	Added digits from left to right. When the sum of a column is greater than 10, the "unit," or "one's" place-holder is carried to the column on the right	Inaccurate procedure; lack of regard for place value[2]

Sources: [1]Miller and Hudson, 2005; [2]Mercer & Mercer, 1998, as cited in University of Kansas, n.d.; MathVIDS, n.d.

Concluding Thoughts

Identifying students’ consistent errors or misconceptions is the first step to providing remedial or corrective instruction. Teachers can typically identify and describe specific errors, particularly in subtraction problems (Riccomini, 2005), however, the next step, selecting appropriate instructional foci, is more challenging. In this step, teachers need to specifically address the students’ particular weaknesses. Teachers seldom focus on the patterns of error, but instead concentrate on just the basic facts (Riccomini, 2005). It is common for teachers to emphasize basic facts instruction when they re-teach or correct students’ errors, overlooking procedural and conceptual knowledge (Babbitt & Miller, 1996; Woodward, Baxter, & Robinson, 1999). Attending to fact errors not only translates to inefficient use of teaching time, it does not help to correct students’ misconceptions or misunderstandings at the conceptual or procedural levels. Some ways to approach this issue include: (1) breaking down instruction into smaller sections so that students are exposed to more explicit instruction on specific parts of concepts or parts of procedural steps, and (2) using curriculum materials and textbooks that include specific suggestions for re-teaching or strategies to help correct students' errors (Riccomini, 2005).

Another important thing to remember when engaging in error analysis relates to student attention. Even though poor attention is one of the plausible reasons why students persistently make errors (Stein, Silbert, & Carnine, 1997), there are concerns that teachers may exclusively look for this trait and fail to consider other reasons (Riccomini, 2005). Teachers should look into other factors like curriculum materials and instructional design and delivery methods (Riccomini, 2005; Stein, Silbert, & Carnine, 1997).

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